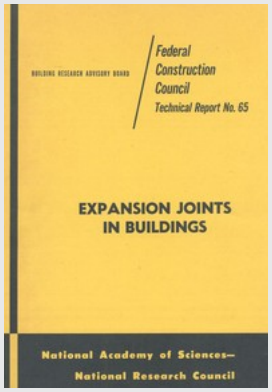

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EXPANSION JOINTS IN BUILDINGS

Technical Report No. 65

Prepared by the Standing Committee on Structural Engineering of the
Federal Construction Council
Building Research Advisory Board
Division of Engineering
National Research Council

NATIONAL ACADEMY OF SCIENCES
Washington, D.C. 1974

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FOREWORD

Many factors affect the amount of temperature-induced movement that takes place in a building and also the extent to which this movement can take place before serious damage will occur or extensive maintenance will be required. Because of the complexity of the problem, no one has yet established nationally acceptable procedures for precisely determining the size and location of expansion joints required in a particular structure. In the absence of such definitive procedures, most designers and federal construction agencies have individually developed guidelines based on rough calculations and experience.

Although relatively few serious problems attributable to inadequate provision for temperature-induced movement have been reported, significant differences are found in the various guidelines used for locating and sizing expansion joints, suggesting that at least some of the guidelines must be in error. Therefore, it is quite likely that in some cases joints are being omitted where they are needed--thus creating a risk of structural failure or causing unnecessary operations and maintenance costs--and in other cases they are being used where they are not required--thus increasing the initial cost of construction and creating space utilization problems.

As a consequence, the Federal Construction Council (FCC) undertook the study reported herein in hopes of developing more definitive criteria for expansion joints than have existed in the past. The study was carried out for the Council by the FCC Standing Committee on Structural Engineering.

This report has been reviewed and approved by the Federal Construction Council, and, on the recommendation of the Council, the Building Research Advisory Board (BRAB) has approved the report for publication.

The Board gratefully acknowledges the work of the FCC Standing Committee on Structural Engineering in conducting the study and developing this report.

HERBERT H. SWINBURNE, Chairman

Building Research Advisory Board

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I

INTRODUCTION

A. PURPOSE OF REPORT

The purpose of this report is to provide federal agencies with practical procedures for evaluating the need for through-building expansion joints in structural framing systems and with guidelines for designing expansion joints for those systems when they are required.

B. SCOPE

The term “expansion joint” as used throughout this report refers to the isolation joints provided within a building to permit the separate segments of the structural frame to expand and contract in response to temperature changes without adversely affecting the building's structural integrity or serviceability.

This report is limited to the investigation of expansion joints that permit movement in the horizontal direction only. All types of building and all climatic conditions experienced in the United States are considered. Problems such as those associated with vertical movement of the structural frame, dimensional changes of cladding, relative motion of cladding to frame, shrinkage of concrete, manufacturing errors in the length of members, and differential settlement of foundations are not considered.

C. CONDUCT OF STUDY

The study on which this report is based was carried out under the direction of the Federal Construction Council Standing Committee on Structural Engineering. The Committee first examined in detail an unpublished report in which horizontal changes in dimension in nine federal buildings were observed and related to recorded temperature changes. Additionally, the Committee studied the current practices of federal agencies regarding expansion joint criteria.

To enchance its understanding of the distribution of stresses and associated deformation in frames subjected to uniform temperature change, the Committee formulated and conducted an analytical study of the effects of uniform temperature change on typical two-dimensional elastic frames. A theoretical computer model was developed for this purpose. Observed dimensional changes caused by temperature changes were correlated with data obtained from the computer analysis. The results of the Committee's study and analysis, as well as its collective experience and judgment, served as the bases for this report.

D. ORGANIZATION OF REPORT

This report includes two main sections in addition to this Introduction: Section II, Recommendations, in which the Committee presents its recommendations without detailed discussion, and Section III, Discussion, in which the Committee presents the data and rationale upon which the recommendations are based and on which attention is focused on the nature of the problems associated with temperature changes and their effects on structural integrity and building serviceability. The computer printout of an elastic analysis that illustrates the effects of temperature changes on horizontal dimensional movements and a compilation of temperature data for various cities in the United States are included as appendixes.

II

RECOMMENDATIONS

A. GENERAL

1. The structural analysis of a building should include a determination of the need for thermal expansion joints in view of the potential impact of temperature-produced dimensional changes on structural integrity and building serviceability.*
2. As a minimum, each of the following factors should be examined and taken into account during expansion joint location and design:

a. Dimensions and configuration of the building.

b. Design temperature change, which should be computed in accordance with the formula:

$\Delta t = (T_w - T_m)$ or $(T_m - T_c)$, (1)

whichever is greater, where,

T_m = the mean temperature during the normal construction season in the locality of the building. For the purpose of this report, the normal construction season for a locality is defined as that contiguous period in a year during which the minimum daily temperature equals or exceeds 32 °F. [For example, the normal construction season for Anchorage, Alaska, is 5-1/2 months (April 24-October 8) and for Birmingham, Alabama, is year-round (January-December).]

T_w = the temperature exceeded, on the average, only 1 percent of the time during the summer months of June through September in the locality of the building. (In a normal summer there would be approximately 30 hours at or above this design value.)

T_c = the temperature equaled or exceeded, on the average, 99 percent of the time during the winter months of December, January, and February in the locality of the building. (In a normal winter there would be approximately 22 hours at or below this design value.)

*Dimensional changes in the vertical direction and methods of fastening nonstructural elements to the structural frame of the building do not fall within the scope of this report.

Values for T_w, T_m, and T_c for different localities in the United States are included in Appendix B. The T_w and T_c values were extracted from the ASHRAE Handbook of Fundamentals (1972) published by the American Society of Heating, Refrigerating and Air-Conditioning Engineers.

c. Provision for temperature control.
d. Type of frame, type of connection to the foundation, and symmetry of stiffness against lateral displacement.
e. Materials of construction.

B. CRITERIA FOR DETERMINING THE NEED FOR EXPANSION JOINTS

The need for thermal expansion joints in buildings may be determined initially on an empirical basis. If results are deemed by the designer to be too conservative or if the empirical approach is not sufficiently comprehensive to be applicable to the type of structure being investigated, a more precise analysis should be undertaken. In either case, the following criteria should be utilized in the absence of more rational approaches.

1. Empirical Approach

a. For buildings having a beam-and-column or slab-and-column structural frame,* the maximum length of the building** without expansion joints should be determined in accordance with Figure 1 on the basis of the design temperature change (Δt) in the locality of construction.
b. For buildings supported by continuous exterior unreinforced masonry, expansion joints should be placed at intervals not exceeding 200 feet. In addition, intermediate subjoints should be positioned and spaced in accordance with the recommendations of the Brick Institute of America (BIA) and the National Concrete Masonry Association (NCMA).***

*A building should be considered to have a beam-and-column or slab-and-column structural frame even if intermittent interior shear walls or other stiffening elements are incorporated in the frame and even if the frame is supported on an above-grade reinforced concrete continuous perimeter base wall. The provisions of this recommendation do not apply to buildings with fully exposed exterior frames placed outside the cladding elements.

**The maximum diameter or diagonal of a round, elliptical, or closed polygonic building should be considered its maximum dimension.

***At the time of this writing such recommendations are provided in the BIA publications, Differential Movement, Cause and Effect (No. 18, April 1963), Differential Movement, Expansion Joint (No. 18A, May 1963), and Differential Movement, Flexible Anchorage (No. 18B, June 1963) and the NCMA publication, Control of Wall Movement with Concrete Masonry (TEK 3, 1972).

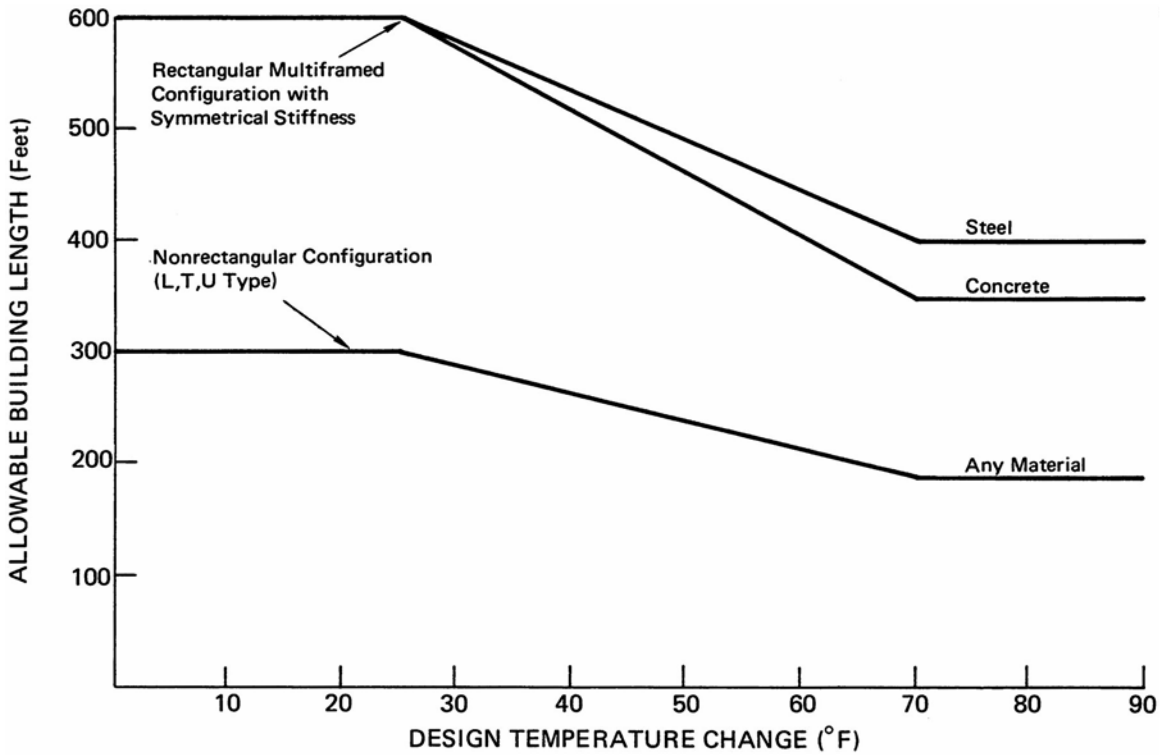


FIGURE 1 Maximum allowable building length without use of expansion joints for various design temperature changes. These curves are directly applicable to buildings of beam-and-column construction, hinged at the base, and with heated interiors. When other conditions prevail, the following rules are applicable:

(a) If the building will be heated only and will have hinged-column bases, use the allowable length as specified;
(b) If the building will be air conditioned as well as heated, increase the allowable length by 15 percent (provided the environmental control system will run continuously);
(c) If the building will be unheated, decrease the allowable length by 33 percent;
(d) If the building will have fixed-column bases, decrease the allowable length by 15 percent;
(e) If the building will have substantially greater stiffness against lateral displacement at one end of the plan dimension, decrease the allowable length by 25 percent.

When more than one of these design conditions prevail in a building, the percentile factor to be applied should be the algebraic sum of the adjustment factors of all the various applicable conditions.

2. Analytical Method

For those situations in which the need for thermal expansion joints cannot be determined on an empirical basis or in which the empirical approach provides a solution that professional judgment indicates is too conservative, a detailed analysis like that discussed in Section III.B.2 should be performed. The analysis should include identification and evaluation of the effects of the factors listed in III.A.2, as well as a stress-strain analysis of the effects on the structural frame of a uniform temperature change, $C\,\Delta t$, where Δt is computed as described under III.A.2.b and the coefficient C is:

a. Equal to unity for unheated buildings,
b. Equal to 0.70* for buildings heated but not air conditioned, and
c. Equal to 0.55* for buildings heated and air conditioned.

C. SUGGESTED PROCEDURES FOR DESIGN OF EXPANSION JOINTS

The following guidelines are recommended as bases for expansion joint design and location:

1. Expansion joints should extend over the entire height of the building from the top of the foundation footing (or perimeter basewall) through the roof. The resulting two separate but adjacent structural frames may share the same footing.
2. The upper bound [UB) of horizontal joint closing in buildings with a beam-and-column frame should be calculated from the expression:

   $UB = 6 \cdot 10^{-6} \cdot \Delta t_e \cdot L$, (2)

   where $\Delta t_e = (T_w - T_m)$ in degrees Farenheit and L = effective length.**

*The C values of less than unity are based on the assumption that the environmental control system in the building would operate continuously. Hence, the lower C value cannot be applied if it is anticipated that the environmental control system will be regularly shut down for extended periods of time (i.e., 2 days or longer). Any deviation from these values should be quantitatively justified.

3. To allow for construction tolerances and compressibility and expandability of the joint sealants, the expansion joint width (W), in inches, should be computed as follows:
   $W = C_1 \cdot UB$, (3)
   where UB is as computed in Eq. (2), and C1* = 2.0 for unheated buildings, 1.7 for buildings heated but not air conditioned, or 1.4 for buildings for both heated and air conditioned.
4. For buildings with continuous exterior bearing walls of clay masonry, the maximum spacing of the expansion joints should be limited to 200 feet, and the minimum required joint width (W), in inches, should be calculated from the following expression:
   $W = C_1 \cdot L\ (50°F \cdot \Delta t_e)\ (4 \cdot 10^{-6})$, (4)
   where Δt_e, C_1, and L are as defined for Eq. (2) and (3).
5. The minimum width of an expansion joint should in no case be less than 1 inch. If the computed expansion joint width exceeds 2 inches, special consideration should be given to the materials and methods of joint construction to ensure that the joint itself will be able to withstand the distress caused by substantial movement at the joint. (Additional consideration should be given to architectural and structural details to ensure that the building will tolerate the inherent deformations without loss of serviceability.)
6. Expansion joint design should permit uninterrupted relative motion of the abutting building segments, prevent the entrance of water or debris, and allow for easy inspection and maintenance.

D. AREAS OF FUTURE RESEARCH

Research directed toward the establishment of a valid data base for the development of technically sound criteria for the design and location of expansion joints should be initiated immediately. Special attention should be given to the following:

1. The collection, classification, and interpretation of data on building damage attributable to temperature fluctuation.

*Coefficient C_1 differs from coefficient C in that C_1 takes into consideration construction tolerances and the compressibility/expandability of the joint filler, as well as temperature.

2. The development of data necessary for the correlation of ambient temperature with temperatures of building components (structural and nonstructural) at the periphery and within buildings for different building types and materials.
3. The development of data for the correlation of ambient temperature fluctuations with temperature gradients existing within building components under different conditions of exposure and types and methods of insulation.
4. Analytic and experimental investigation that will lead to the correlation of stresses in the various building components to the different patterns of temperature fluctuations and gradients and to the different types of assembly component (connectors).

The effects of temperature change on the performance of buildings supported on masonry walls should be examined for each type of masonry material or combination of materials likely to be used, and each type or combination of materials should be investigated with respect to construction details, connections of walls to horizontal and vertical components (roofs, floors, walls, and partitions at right angles), optimal spacing of joints, and extent of joints.

III

DISCUSSION

A. GENERAL

A building is a dynamic product subjected to a host of conditions that keep its various elements in a constant state of stress, strain, and displacement. During design, displacement must be evaluated and, when necessary, controlled to ensure that the building will perform as intended throughout its expected life without the need for unanticipated large-scale maintenance. Expansion joints introduced by the designer to avoid the effects of large lateral displacements are relied upon to limit the internal stresses caused by expansion and contraction and the actual movement of building elements, permit relative motion of the building members without disturbing functional continuity, and affect a complete structural separation without disturbing structural integrity.

Experience indicates that appropriate use of expansion joints presents a rather complex design problem and requires a thorough understanding of those factors that dictate their need as well as those that affect their ultimate performance after installation. The design, location, and performance of expansion joints can be influenced by such factors as building form, function, and economics; construction techniques; the varying characteristics of the different materials employed, changes of these characteristics under varying environmental conditions, and the physical relationship of one to the other; and the ability to withstand stresses resulting from dimensional changes.

The problem is further complicated by recent trends and developments in structural engineering. A better understanding of the behavior of materials and an evolution in the precision of structural analysis of buildings, coupled with the advent of computers that permit economical, rapid, and accurate analyses, have encouraged designers to include a mixture of materials and a variety of jointing systems in most major structures. These factors make it possible to generally decrease the dimensions of resisting structural elements from those customarily used in past practice. As a consequence, structures are less likely to be overdesigned than in the past; therefore, the risk of their reaching the threshold of structural failure is greater, giving emphasis to the importance of adequate expansion joints.

Previously developed empirical rules for expansion joint spacing are not necessarily compatible with these recent trends and developments. If desired margins of safety are to be maintained, it appears that the need for thermal expansion joints should be determined as part of the structural analysis of a building and that special attention should be given to the potential impact of horizontal dimensional changes* on structural integrity and building serviceability. Factors that are considered to be most significant with respect to the design and location of expansion joints, and which are treated herein, include dimensions and configuration of the building; design temperature change; provision for temperature control; type of frame, type of connection to the foundation, and symmetry of stiffness against lateral displacement; and materials of construction.

1. Dimensions and Configuration of the Building

The dimensions of a building are obviously an overriding parameter with regard to the need for expansion joints because the problem of expansion joints arises when the dimensions become substantial.

The configuration of a building is a parameter influencing the severity of the effects of temperature changes on a building and, as such, should be given consideration during the design process. Rectangular buildings and buildings with two axes of symmetry in plan with no internal open courts experience temperature-induced stresses that have relatively simple patterns, while buildings with a more complex configuration, such as U-shaped or L-shaped buildings, experience horizontal dimensional changes that result in complex stress patterns, particularly at re-entrant corners.

2. Temperature Change

Since construction is carried out over a considerable period of time, the various elements of the structure are installed at different temperatures. The temperature changes causing displacements and stresses in a structure are changes from these installation/erection temperatures, over which the designer has little, if any, control. Yet, while it is apparent that temperature change is one of the most important factors influencing the potential linear expansion/contraction of a building, there is no possibility of establishing exactly the maximum expected temperature change because this change is not the same for all parts of the structure and is not known during the design phase for any one particular part of the structure.

a. Computation of design temperature change

To properly account for the effects of temperature on buildings requires a procedure that uniquely defines the temperature differences for which a building in a given locality should be designed. Currently, however, there is no one established procedure for determining this design temperature change with precision; therefore, the

*While the scope of this report is concerned only with horizontal dimensional changes, the analysis also should be supplemented by consideration of dimensional changes in the vertical direction of buildings and of methods used to fasten nonstructural elements to the structural frame of the building.

following guidelines were developed by the Committee to serve as an aid in its computation until a more definitive procedure is developed:

- It should be assumed that structures will be built when the minimum daily temperatures are above 32 °F.
- Mean temperatures (T_m) should be based on only the construction season--the contiguous period* during which the minimum daily temperatures are above 32 °F. This season varies for different localities (see Table 1) and, except for southern areas, the mean construction season temperature is different from the mean annual temperature.
- The anticipated high-temperature extreme (T_w) should be considered as the temperature that is exceeded, on the average, only 1 percent of the time during the summer months (June through September) in the locality of the building.
- The anticipated low-temperature extreme (T_c) should be considered as the temperature that is equaled or exceeded, on the average, 99 percent of the time during the winter months (December through February) in the locality of the building.
- Using the data described above, the design temperature change (Δt) can be uniquely defined according to $\Delta t = (T_w - T_m)$ or $(T_m - T_c)$, whichever is greater. The T_w, T_m, and T_c values for many localities in the United States are presented in Appendix B.

TABLE 1 Mean Construction-Season Temperatures for Various Localities

	Construction Season			
Locality	From	To	Mean Temperature (°F)	Annual Mean Temperature (°F)
Birmingham, Alabama	Jan 1	Dec 31	63.2	63.2
Anchorage, Alaska	April 24	Oct 8	50.6	35.5
Almose, Colorado	May 8	Sept 28	60.4	42.2
Daytona Beach, Florida	Jan 1	Dec 31	70.3	70.3

*Obtained from Decennial Census of United States Climate--Daily Normals of Temperature and Heating Days, Climatography of the United States No. 84, U.S. Department of Commerce, Weather Bureau, Washington, D. C. (1963).

b. Differential temperature effects on a building element

As illustrated in Figure 2, the differential temperature profile of a member can be assumed to consist of the superposition of two temperature profiles:

(1) A uniform temperature change (Δt_g) equal to the temperature change that takes place along the axis of the member, and

(2) A differential temperature change [$d(\Delta t)$] equal to the difference of the temperature change at one face of the member less the temperature change at the opposite face of the member; i.e., $d(\Delta t) = (\Delta t_2 - \Delta t_1) = (a+b)$.

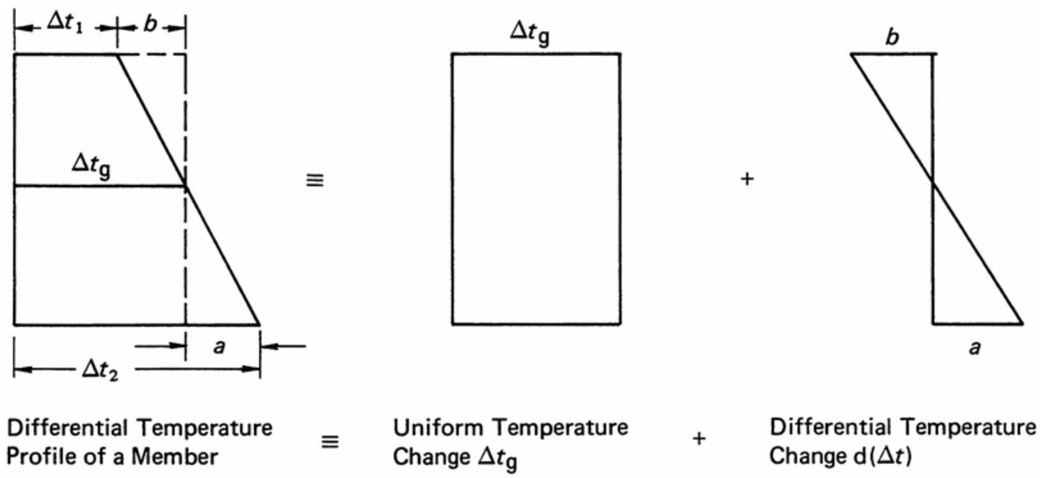


FIGURE 2 Differential temperature effects on a building element.

When viewed in this manner, it becomes apparent that the differential temperature change [$d(\Delta t)$] causes no change in the length of the member along its axis. Instead, it tends to cause curvature in the member, which, to the extent it is resisted, results in internal stresses. However, neither the curvature nor the ensuing internal stresses propagate and cause a cumulative increase in the length of the structure as do those stresses and deformations brought about by uniform temperature change.

Thus, with respect to expansion joint requirements, a differential temperature profile can be replaced by the superposition of a uniform temperature change corresponding to the change at the level of the centroidal axis of the member and a differential temperature change that causes no change in the overall length of the member.

From this discussion it becomes evident that the uniform temperature change component (Δt) of the differential temperature profile is the principal cause of building distress due to temperature changes. For a symmetrical member, the effective uniform temperature change will be equal to the average of the temperature changes on the opposite faces undergoing differential temperature changes. In members with nonsymmetrical cross sections, the effective uniform temperature change obviously will have an intermediate value between the temperature changes on the two opposite faces.

3. Provision for Temperature Control

   Properly functioning heating and air conditioning in a building will maintain a relatively constant temperature within the building and, thus, reduce the potential for adverse temperature change effects on internal and peripheral members. However, buildings that are heated but not air conditioned are subject to substantial changes in temperature during the summer and these must be taken into account. Buildings that are both heated and air conditioned can be considered only theoretically immune to the effects of extreme temperature fluctuations since malfunctions or intentional shutdowns of mechanical equipment could lead to sudden injurious temperature variations. Thus, the potential for such also must be considered during expansion joint design.

4. Type of Frame, Type of Connection to the Foundation, and Symmetry of Stiffness against Lateral Displacement

   Thermal effects on buildings with fixed-column bases are likely to be more severe than on buildings with hinged-column bases. Comparison of the behavior of two identical tall buildings, one with fixed-column bases and one with hinged-column bases, subjected to the same temperature changes indicates that both buildings underwent virtually the same dimensional changes in all levels above the first. However, in the case of the fixed-column building, temperature-induced stresses (shear forces, axial forces, and bending moments) at critical sections within the lowest story were almost twice as high as those at corresponding locations in the hinged-column building.

   The extent of stresses and deformations in a building also will be greatly influenced by the symmetry of the building in terms of stiffness against lateral displacement. A building with main structural frames having approximately the same stiffness against horizontal displacement from the center to the right as from the center to the left will be subject to smaller stresses and deformations than a similar building with main structural frames having columns or a shear wall at one end substantially stiffer against horizontal displacement than the rest of the columns. Therefore, the design of expansion joints should be influenced by the type of frame, type of connection to the foundation, and the stiffness against lateral displacement of the structural framing, each of which is discussed in greater detail subsequently.

5. Materials of Construction

The type of material used in the construction of the frame (e.g., steel, concrete, masonry) can influence the effects thermal changes will have on the building. For example, comparison of the effect of thermal changes in two similar frames with identical moments of inertia, one of which has beams with greater cross-sectional area than the other, indicates that the frame with the greater cross-sectional area develops the greater axial forces, shear forces, and bending moments at the critical sections of both beams and columns. Consequently, considering that the ratio of the cross-sectional area to the moment of inertia is greater for concrete frames than for steel frames, it would be reasonable to expect that the thermal effects on structures would generally result in higher stresses in concrete frames than in steel frames. Therefore, a designer would need to be somewhat more conservative in evaluating potential thermal effects when using concrete as a structural material than when using steel, unless of course, he conducts a more complete analysis of the structure for all forces, including thermal effects, and provides explicitly for the critical loading conditions.

Shrinkage of concrete members accounts for a portion of the dimensional change in a building frame. However, shrinkage usually takes place during a relatively short period of time following concrete placement. Its extent can be estimated with reasonable accuracy, and its effects can be controlled by proper planning of the construction sequence of the building. Consequently, concrete shrinkage has not been considered in this study but should be taken into account when planning the construction sequence of concrete frames of lengthy buildings.

B. DETERMINATION OF NEED FOR EXPANSION JOINTS

Although the need for expansion joints can be determined empirically in many cases, in certain situtations it would be determined best through analytical evaluation. The empirical approach is likely to be the simpler of the two, but is the more conservative. The analytical method requires that the designer fully evaluate and account for in the overall design the effects of all factors influencing the need for expansion joints (discussed in Section III.A). The basic elements, concepts, and/or procedures involved in each are discussed below.

1. The Empirical Approach

a. Current practices and existing data

With the exception of criteria currently used by some federal agencies in determining building expansion joint requirements and a report concerning a one-year (1943-1944) experiment examining expansion joint movement, a search of the literature revealed no significant quantitative data or specifications. Considerable information on the design of the actual expansion joint for a variety of specific building components and materials is available in engineering/architectural aids and specifications; however, through the years, the decision concerning the number and location of expansion joints, as well as the ultimate design, has been left primarily to the judgment of the designer on the basis of his intuition and experience. Individual agencies have examined the performance of buildings that seemed to lack appropriate expansion joints and have provided remedies for such problems on a case-by-case basis. Unfortunately, the results relating cause and effect were not formally documented.

An examination of the federal agency criteria for expansion joints indicates that they are very basic in concept. These criteria are based on the assumption that the maximum allowable linear dimension of buildings is a function of two parameters:

(1) The maximum difference between the mean annual temperature at the locality of the building and the maximum or minimum expected temperature, and
(2) The provision for heat control in the building under consideration.

The first parameter causes the dimensional change, while the second reflects the ability of the building to dampen, and thus to reduce, the severity of the effects of outside temperature changes. Curves for heated and unheated buildings (Figure 3) are used to relate the maximum allowable length of a building without expansion joints as a step function to design temperature changes.

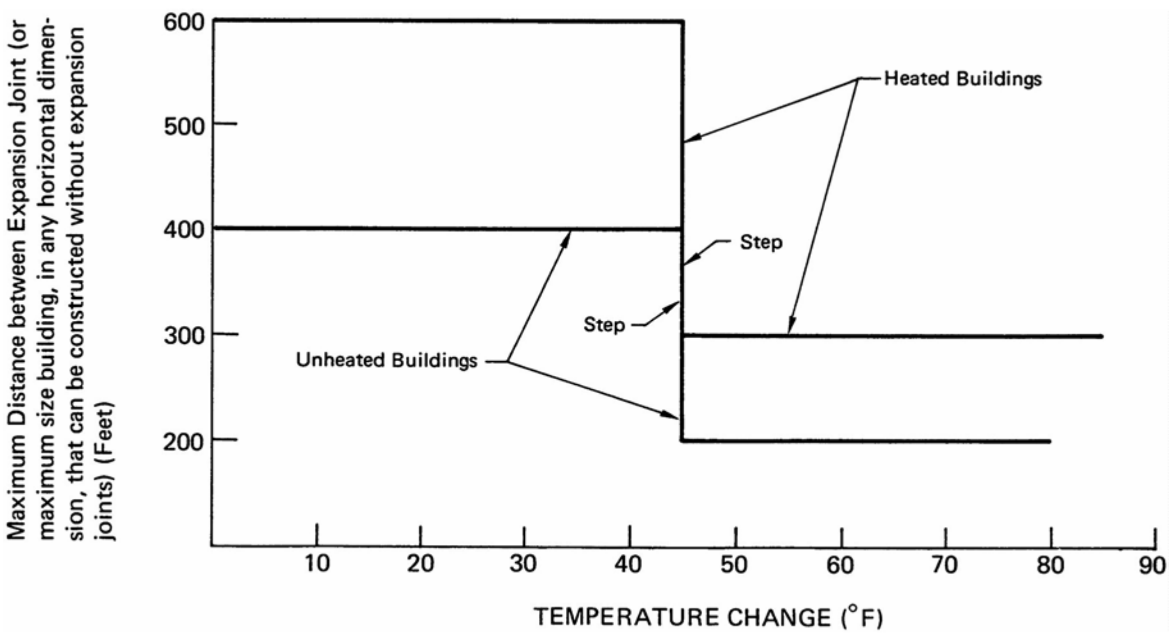


FIGURE 3 Expansion joint spacing criteria of one federal agency.

There is little doubt that a step function cannot represent the behavior of a physical phenomenon, such as thermal effect, that has evident characteristics of continuity. However, while the maximum allowable building length can be expected to decrease as the design temperature change increases, the definition of the exact nature of their relationship requires more rigorous and elaborate quantitative data than is available at present or is expected to become available. Therefore, the limits of 600 and 200 feet in the linear dimensions of buildings are assumed to reflect the considered consensus of long experience within the engineering profession. Consequently, without any further experimental or theoretical justification, they are used herein as boundary values.

Taking the above factors into consideration, the curves in Figure 1 have been developed and are recommended in Section II as an aid in the empirical determination of the need for expansion joints in buildings. These curves are within the 600- and 200-foot bounds and assume a linear change (in the absence of any evidence justifying curves of other shapes) in allowable maximum length with regard to design temperature change. For relatively small temperature changes (up to 25 °F) the maximum allowable length is permitted. In addition, factors can be used to modify the maximum allowable building lengths obtained from Figure 1 for parameters other than heating (e.g., air conditioning, type of support, type of configuration, and type of material used) to account in a conservative manner for their influence. These factors were listed in Section II, Recommendation A.2 , and are based on a qualitative assessment; the following sections of this report provide the rationale for their adoption.

b. Findings of a previous study on expansion joints

Structural engineers of the Public Buildings Administration* investigated expansion joint movement over a period of one year (September 1943 to August 1944) in nine federal buildings to obtain measurements of dimensional changes over a complete cycle of seasons.** Although some of the assumptions made in the analysis are questionable and data collected are not sufficiently complete to serve as a basis for definitive statements, several important conclusions relative to this report can be drawn from that investigation and those considered most significant are presented below.

(1) There is a considerable time lag (2 to 12 hours) between the maximum dimensional change of a building and the peak ambient temperature associated with this dimensional change. The investigators theorized that the time lag was due to the temperature

*Now the Public Buildings Service of the General Services Administration.

**Public Buildings Administration, Movement of Expansion Joints in Nine Federal Buildings in Washington, D. C. (September 10, 1943-August 29, 1944), unpublished.

gradient between the outside ambient temperature and the inside temperature of the building, the resistance to the transfer of temperature change (insulation), and the duration of the ambient temperature at its extreme levels. Since such parameters determine the rate of temperature change at the axis, this theory appears to be valid.

(2) The maximum temperature change and the maximum linear dimension of a building are not the only parameters affecting the extent of dimensional change in the building. For example, the effective coefficients of thermal expansion appear to vary widely from building to building and even within a single building.

(3) The effective coefficient of thermal expansion of the first floor level is approximately one-third to two-thirds that of the upper floors.

(4) The dimensional change of each building at the upper level corresponds, in most cases, to an effective coefficient of thermal expansion between 2 and 5 per million degrees Fahrenheit. Given the value for this coefficient of 3.3 for brick, 5.5 for concrete, and 6 for steel and the uncertainty of the assumption used to evaluate the temperature change on the basis of which the range from 2 to 5 was derived, the investigation seems to confirm that the upper levels of buildings undergo dimensional changes corresponding to the coefficient of thermal expansion of the principal material of which each is constructed.

c. Explanation of structural expansion by statics

The problem of structural expansion due to temperature change can best be understood in light of the basic mechanics involved. A fundamental analysis of the problem can be made by utilizing statics. Assume for this purpose a one-bay simple bent (Figure 4), free in the two-dimensional space and subjected to uniform positive temperature change. Intuitively, it becomes obvious that the bent ABCD will expand as shown in Figure 4ato the new configuration A`B`C`D` with no accompanying stresses since the expansion is completely unrestrained. If bent $A_1B_1C_1D_1$ is fixed at the ground, expansion of the bent will occur as shown in Figure 4b. In its deformed position ($A_1B`_1C`_1D_1$) the bent will be under stress. Supports A_1 and D_1 will develop horizontal thrust H_1 and fixing moment M, thus subjecting beam B_1C_1 to an axial compressive force. Due to this internal force, the expansion $B_1B`_1$ of Figure 4b will be smaller than BB` of Figure 4a. If, on the other hand, the bent is hinged at the bottom, as shown in Figure 4c, there will be no support moment and horizontal thrust H_2 will be smaller than H_1. As a result, the compression of $B`_2C`_2$ will be smaller than that of $B`_1C`_1$. The elongation, $B_2B`_2$, of Figure 4c therefore will be greater than $B_1B`_1$ of Figure 4b but smaller than BB` of Figure 4a. Similarly, the compression of $B`_2C`_2$ will be between the zero compression of BC and the compression $H_1$ of $B`_1C`_1$

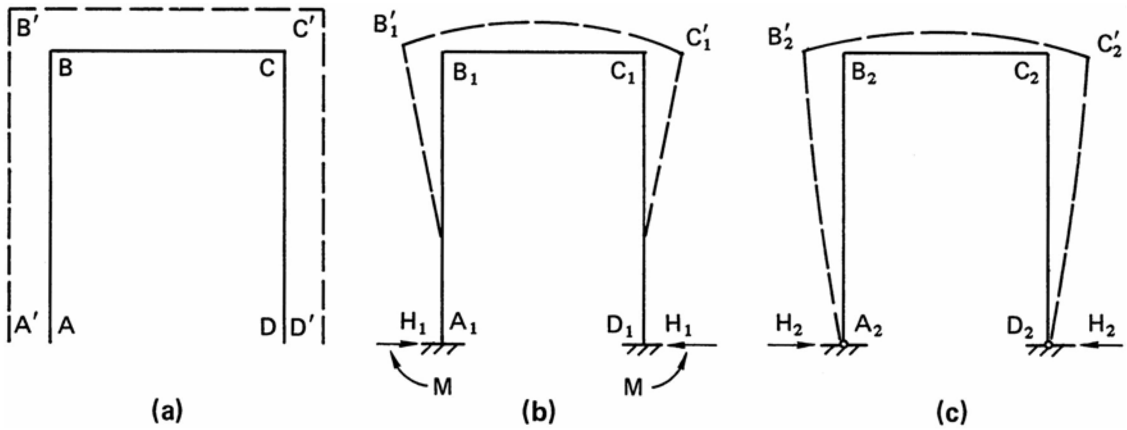


FIGURE 4 Analysis of one-bay simple bent subjected to uniform temperature change: (a) bent completely unrestrained; (b) bent fixed at ground; (c) bent hinged at ground.

In the multistory and multibay frame conditions illustrated in Figure 5, a temperature increase will produce a pattern of stresses and deformations similar to those of the single bent of Figure 4. Although it is more difficult to visualize the mechanics, it remains possible to predict the relative intensities of the thrusts and horizontal joint movements. Due to symmetry, the intensity of the thrusts (H_1, H_2, ... in Figure 5a and $H`_1$, $H`_1$, ... in Figure 5B) is maximum at the extreme ends and approaches zero at the center. Similarly, the horizontal displacements of the joints within each floor are maximum at the ends and approach zero at the center of the frame.

These progressive changes of magnitude are a result of the cumulative effect of elongation from the center to the outside. It can be reasoned that the beams near the center of the frame are subjected to maximum axial stresses while the columns near the edges of the frame are subjected to maximum bending moments and shear forces. However, the intensity of these forces, and the accompanying elongations, may vary from story to story and their assessment will require analytical study.

d. Analyses of stresses and deformations in frames

An analytical study was formulated and conducted by the Committee to investigate the effects of uniform temperature change on typical two-dimensional elastic frames. It was anticipated that, with the aid of a computer program for two-dimensional stress analysis, the study would facilitate the understanding and evaluation of the temperature effects on joint displacement and forces (shear, axial, and bending) in a long building.

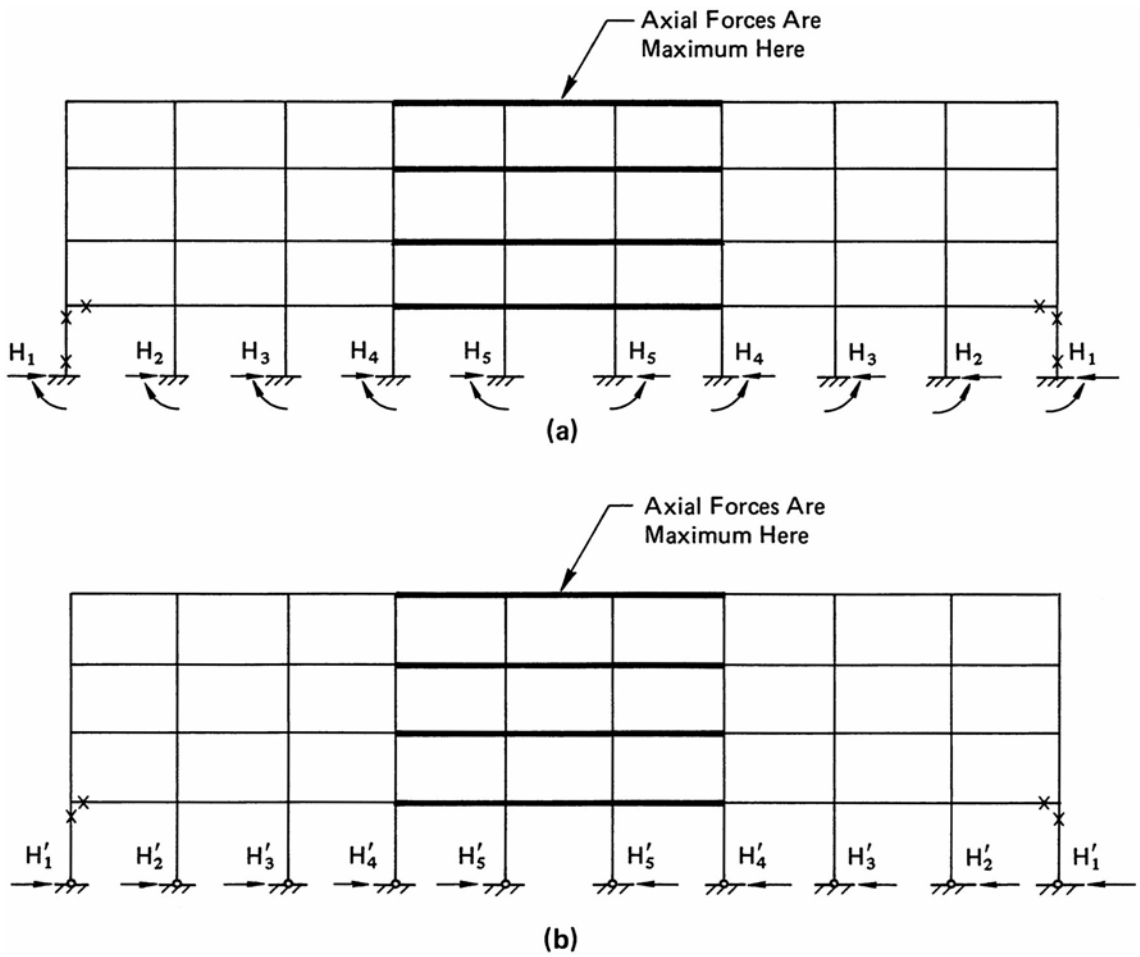


FIGURE 5 Analysis of multistory and multibay frame subjected to uniform temperature change (x = points of maximum bending moments and maximum shear forces): (a) frame fixed at ground; (b) frame hinged at ground.

It is recognized that, in the analysis of a building frame, the actual moments, shears, and thrusts that occur at any point depend on the degree of fixity obtained in the structural joints, the degree of fixity assigned to the foundation of the structure, and other pertinent considerations of this nature. However, for reasons of simplicity and in order to gain a basic qualitative understanding of the nature of temperature effects on a typical frame, extreme conditions were utilized for such considerations during the computer analysis.

The frames analyzed were given a typical bay of 25 feet and a height of 13 feet for the first story and 10 feet for the stories above. These dimensions were common in all frames; however, the following conditions (Table 2) were differentiated:

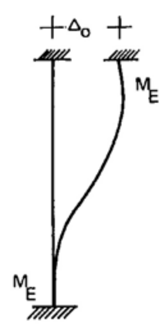
(1) Columns fixed at the base (e.g., Anal. 1-1) or columns hinged at base (e.g., Anal. 2-1).
(2) Either 24 x 24-inch (e.g., Anal. 1-1) or 16 x 16-inch (e.g., Anal. 3-1) columns framed to 14 x 20-inch beams.
(3) Frames with two (e.g., Anal. A-1) or three stories (e.g., Anal. 4-1) comprised of columns and beams with identical dimensions.
(4) Frames with beams of a given moment of inertia but different cross-sectional areas (e.g., Anal. A-2)
(5) Frames with eight bays of 25 feet each (e.g., Anal. 1-1) and a similar frame with 16 bays of 25 feet each (e.g., Anal. B-1).
(6) Frames symmetrical about a vertical axis and frames with the extreme columns on one end substantially stiffer than the rest of the column (e.g., Anal. M-1; see Appendix A).
(7) Frames with all columns fixed to the beams at their upper end and frames with outside columns hinged both at top and bottom (e.g., Anal. M-2)

All frames were analyzed for a 100 °F uniform temperature increase and for a coefficient of thermal expansion equal to 6 per million degrees Fahrenheit. An example of the computer output of this study (Anal. M-1) is presented as Appendix A. The tabulated results of the analysis are shown in Table 2.

Column 11 of Table 2 lists the horizontal displacement ratio Δ/Δ_o for the oudside joint of the lowest story of the analyzed frames, where Δ_o is the unrestrained displacement of a joint and Δ is the actual displacement of the joint. A review of analyses 1-1 and 2-1 verifies that for similar frames with fixed or hinged columns Δ/Δ_o is 0.71 and 0.87, respectively. It is noted that the hinged column

TABLE 2 Tabulated Results of Analysis

Anal. No.	Col. Size (in.)	Beam Size (in.^{2})	Base Type	Total Length (ft)	No. Stories	Beam Max. M^a	Beam Max. N^a	Column Max. M^a	Column Max. V^a	Δ/Δ_o (%)[b]	M/M_E (%)[b]	V/V_E (%)[b]	Remarks
1	2	3	4	5	6	7	8	9	10	11	12	13	14
1-1	24	280	F	200	3	70	135	600	69	71	48	36	
2-1	24	280	H	200	3	45	77	250	19	87	21	10	
3-1	16	280	F	200	3	53	70	170	22	89	71	60	
4-1	16	280	H	200	3	25	25	75	6	90	31	15	
A-1	24	280	F	200	2	43	182	570	67	71	46	35	
A-2	24	1000	F	200	2	70	321	750	95	86	60	50	
B-1	24	280	F	400	2	128	280	840	90	55	34	23	
M-1	24	280	F	200	3	100	253	740	83	93	59	44	Stiff End
M-2	24	280	F	200	3	50	138	500	60	83	40	31	Hinges Top & Bottom



[a] All figures rounded.

[b] Δ = displacement at edge of base floor

M = max. column B.M. in inch-kips

V = max. column shear in kips

N = axial force in kips

Δ_o = unihibited displacement at edge of floors corresponding to 100 °F and length of analyzed frame

$= \alpha\Delta_t L = 100°F\alpha L$

$M_E = \frac{6EI\Delta_o}{L^2}$: end moment corresponding to relative displacement Δ_o of ends of member with no rotation permitted at at either end of member

$V_E = \frac{12EI\Delta_o}{L^3}$: shear force developing in member with fixed ends which are displaced relative to each other by Δ_o

has a horizontal displacement ratio less than the unity value corresponding to an unrestrained frame but greater than the 0.71 value for a fixed column. If columns are hinged ra $\frac{(87-71)}{71} \simeq 23\%.$ e base, the maximum deformation of the frame at the first floor increases by approximately

Comparison of analyses 1-1 to 2-1 and 3-1 to 4-1 in columns 7, 8, 9, and 10 of Table 2 reveals that the maximum forces associated with the fixed-column and the hinged-column cases vary in the following ratios:

	For 24 x 24-in. Columns	For 16 x 16-in. Columns
For beam moments	(70-45)/45 = 55%	(53-25)/25 = 110%
For beam axial forces	(135-77)/77 = 75%	(70-25)/25 = 180%
For column moments	(600-250)/250 = 140%	(70-75)/75 = 130%
For column shears	(60-19)/19 = 150%	(22-6)/6 = 250%

Analyses of the results of the various computer runs (Table 2) allow the following observations to be made:

The horizontal displacements (Δ) of all stories except the lower one is almost identical to the displacement, Δ_o, that would develop in a totally unrestrained frame (i.e., Δ` Δ_o). Therefore, if both ends of a frame are equally free to displace, the horizontal displacement of the outside joints of upper stories will be equal to one half of the unrestrained elongation of the frame corresponding to a temperature change, Δt, and a coefficient of thermal expansion, α; that is,

$$\Delta_o = \alpha\Delta_t(1/2L) = 1/2\alpha(\Delta_t)\ L, (5)$$

where α = coefficient of thermal expansion, Δ_o = horizontal displacement of a joint at a distance 1/2 L from the center of the frame, and L = total length of the frame.

In a frame that is restricted from side displacement at one of its two ends, the unimpeded horizontal displacement of the other end will be equal to

$$\Delta_o = \alpha(\Delta_t)\ L, (6)$$

since the total expansion of the full length, L, of the frame will be reflected in displacement of only the unrestricted end of the frame.

A comparison of the data obtained in analyses A-1 and A-2 indicates that for a given frame an increase in the relative cross-sectional area of the beams (not associated with a simultaneous increase in the moment of inertia of the beams) results in a substantial increase in the deformation of the first floor as well as the maximum forces developed in the frame. This is based on the fact that in temperature-induced stressing a force resulting from the structural restraints and the temperature change is proportional to the cross-sectional area of the restrained members (in this case the beams since the frame is not restrained in the vertical direction). Considering that the rate at which cross-sectional areas increase for a given increase in moment of inertia is faster in concrete members than in steel members, it can be anticipated that a concrete frame will suffer somewhat more than a steel frame from the consequences of thermal expansion.

Finally, a comparison of the results of analyses 1-1 and M-2 indicates that hinges placed at the top and bottom of the exterior columns of the frame reduce the maximum stresses that can be expected to develop in the frame. However, such an arrangement permits an increase in the horizontal expansion of the first floor because it reduces the resistance to such movement.

These analytical studies of temperature effects have quantitative value in the sense that they provide valid bounds of stresses and deformations caused by temperature changes and help to define relative values of stress and deformation among the various locations of a structural frame for given ranges of temperature change.

2. The Analytical Method

The difficulties of categorizing every conceivable building configuration and the complexity of the stress and deformation patterns created by thermal change effects in buildings with other than a rectangular configuration make it impracticable to always determine the need for expansion joints on an empirical basis. Also, the designer may wish to exceed the limits on lengths of a building without expansion joints established by the empirical approach described above. In all such cases a detailed structural analysis needs to be performed to support the design. The analysis should incorporate the following basic concepts of and procedures for the design of buildings against the effects of thermal change, regardless of building type or configuration.

a. Uniform design temperature change ($C\Delta_t$)

When establishing for design purposes the effective maximum temperature change to which a structure is likely to be subjected, the influence of heating and air conditioning must be considered as well as the extreme range of outside temperature. However, there are no available experimental or theoretical data or procedures that will permit the quantitative evaluation of the influence of heating and air conditioning in reducing the effects of outside temperature fluctuations on a structure. Even if such could be quantified, only a portion of the dampening effect of temperature control on temperature fluctuation could be recognized safely during frame design in view of the lack of temperature control during the construction phase and during periods when the heating/air-conditioning equipment is likely to be inoperative because of mechanical failure or service and maintenance operations. For these reasons, the calculation of the design temperature change for heated and/or air-conditioned buildings should include a minimum empirical coefficient that will reduce the maximum temperature change to which the structure is expected to be exposed but will not give full value to the influence of internal temperature control. In the absence of technically sound data that dictate otherwise, the uniform design temperature change, $C\Delta_t$, can be satisfactorily determined by considering $\Delta_t = (T_w - T_m)$ or $(T_m - T_c)$, whichever is greater, and C = 1.0 for buildings not provided with temperature control, 0.70 for buildings heated but not air conditioned, and 0.55 for buildings heated and air conditioned. Any deviation from these values should be quantitatively justified.

b. Suggested procedures for design of buildings against thermal changes

As in most structural problems, the investigation of thermal effects on a building is reduced to a basic understanding of distributed forces and deformation within the structure. If deformations are resisted the resulting force system in structural members may well exceed the members' strength and cause structural failure; if they are not resisted the change of geometry in the structure may interfere with its overall performance. Therefore, the designer's task is to select one of the following three broad but basic approaches:

(1) Limit the potential for deformation in the structure (without causing failure) by designing the appropriate members to be substantially stiffened and strengthened.

(2) Allow for substantial movement of the building's structural members and nonstructural components such that ultimate building performance will not be adversely affected. Such a structure will require practically no additional strength of members to withstand thermal effects.

(3) Strike a compromise between capacity to resist stress and ability to withstand deformation without sacrificing building performance.

The first approach is quite unrealistic for buildings above two stories. Stiffening and strengthening the lower floors will only transfer the adverse thermal effects to the stories above, and, in effect, the upper floors would then be resting on a rigid artificial base instead of on the ground. Conversely, this approach is inherent in low, long and massive masonry buildings that have a low tolerance to movement. Their structural frames are designed to maintain building integrity by withstanding the substantial thermal forces that challenge structural strength rather than deformation. Most field experiences indicate that buildings with continuous masonry bearing walls should be provided with expansion joints at intervals not exceeding 200 feet and with additional subjoints in accordance with the recommendations of the Brick Institute of America and the National Concrete Masonry Association.*

The portions of walls at and near the intersection of two walls, surfaces likely to be weakened by numerous openings for doors and windows, and the rigid connections between horizontal elements (particularly concrete or other stiff roofs) and massive walls are most sensitive to the effects of thermal change. In all such cases either expansion joints or very strong elements that can successfully resist the tendency to deform without yielding must be provided. The forces assumed to be generated under these circumstances can be derived by analyzing the forces necessary to cause elastic deformations comparable to the deformations caused in an unrestricted structure by corresponding temperature changes. Thus, these forces can be determined by the very elementary formula:

$$F = \alpha t E A, \quad (7)$$

where F = axial force that develops in a member when it is restrained from changing to temperature change, α = coefficient of thermal expansion, E = modulus of elasticity, A = cross-sectional area, and t = temperature change.

If the member is completely restrained, F will become the maximum axial force which can develop in a member. However, if the member is completely free to expand, F will be equivalent to zero. In actual structures the completely restrained and completely unrestrained conditions are unattainable. Physically, the problem can be interpreted through two superimposed conditions.

Thermal changes cause a total change of length, Δlt, given by the equation:

$$\Delta lt = \alpha t L, \quad (8)$$

where L = the affected length of the member.

Forces resisting the change of length will cause a change in length, Δlf, in the direction opposite Δlt; Δlf is given by Hooke's Law:

$$\Delta \ell_f = \frac{FL}{EA}. \quad (9)$$

*See p. 4 for publication references.

The net change of length will be:

$\Delta l = (\Delta l_t - \Delta l_f)$; (10)

therefore, if $\Delta l = \Delta l_t$ (i.e., $\Delta l_f = 0$ or unrestricted change), then $F = 0$ and if $\Delta l = 0$ (fully resisted change), $\Delta l_f = \Delta l_t$ or $F = \alpha tEA$.

In all real situations F therefore lies between these two extremes; i.e., $0<F<\alpha tEA$.

If $\Delta l = \beta \Delta l_t$, where $0<\beta<1.0$, then $\Delta l_f = \Delta l_t - \Delta l = \Delta l_t\ (1-\beta)$ or $FL/EA = (1-\beta)\alpha tEA$. In interpreting this expression it is observed that if β is the fraction of unrestricted change of length that the member undergoes, the restricting force will equal the complement of this fraction (1-β) multiplied by the absolute restrictive force ($F = \alpha tEA$). Therefore, the designer must first evaluate what percentage of deformation ($\beta \Delta l_t$) the structure can tolerate without loss of performance and then provide for extra strength in the affected members, beyond the requirements of conventional design needs, to resist the forces (1-β) and αtEA. If this cannot be effected within reasonable dimensional or cost constraints, the designer must consider the following alternatives in modifying the structure:

(1) Provide appropriate connections among structural, parastructural, and nonstructural components that will allow a greater tolerance of deformation without loss of performance (i.e., increase β).

(2) Provide for an expansion joint (also called an "isolation" or "separation" joint) in the structural frame, thus reducing the effective length (L) value that influences the primary parameter $\Delta l_t = \alpha tL$.

While the above is a rational and broad approach to investigation and design, one aspect of the interpretation not readily obvious requires special attention. That is, although a particular structure can tolerate unrestricted change of length (which would correspond to β = 1 and F = 0), the designer should evaluate how much of the change of length actually can take place under the physical details of the structure. Such an evaluation is important because the statical redundancy of the structure will resist changes in length, even though the structure can tolerate such changes without loss of performance. Unfortunately, the evaluation of resisting forces, and hence the stresses, that will develop in each situation cannot be effected on the basis of a simple mathematical expression. It will require a thorough knowledge of both structural theory and the details of construction and materials involved. When large and multi-indeterminate structures are involved, a computer analysis may be required to supplement the designer's understanding of the nature and distribution of forces. It is important that, initially, the maximum tolerable β be evaluated and used for design; however, the same procedure then should be applied to the probable maximum resisting forces.

It should be noted that the maximum resisting force will depend not on the maximum tolerable fraction β, but rather on the fraction β that will develop as a result of the physical restraints and redundancy of the structure. This value, β, may well be substantially smaller than the maximum a structure can tolerate. The discrepancy between β (tolerable) and β (developed) is easily recognized in the low-level massive masonry building that can tolerate a great deal of elongation but is so rigid and monolithic that the β that does develop is a small fraction of the value that could be tolerated. The result is a buildup of very high internal forces (F) that can produce failures at the weak points of the structure. These failures are normally brittle in nature, indicating that they were caused by forces exceeding capacity rather than excessive deformation. In such cases the designer has few options for adapting his structure, which is inherently too stiff, and instead, he must design to allow expansion by reducing the effective length (L) that determines the fundamental parameter, $\Delta lt = \alpha tL$. The lowest allowable value of maximum L obviously depends on the maximum expected temperature change (t) since the elongation will be proportionate to both L and t. Conversely, when considering flexible buildings with a frame consisting mostly of slender flexural members, the designer can influence the β (developed) value (i.e., the amount of the maximum change of length that will develop in a building). This can be done by an interplay of strength and flexibility. In general, strong but slender flexible members will allow greater changes of length [i.e., higher β (developed) values], and in these situations, β (developed) will approach the ultimate deformation tolerable to the structure. This combination will ensure that the building will perform with a minimum of distress due to temperature change.

C. THE DESIGN OF EXPANSION JOINTS

The following principles are considered basic to sound expansion joint design.

1. The width of the expansion joint should exceed the maximum potential dimensional changes by an amount sufficient to prevent the complete closing of the joint and, simultaneously, provide for construction tolerances and nature of filler material. The maximum potential dimensional change can be computed either empirically (point 2 below) or by using the formulas given for Δl_t and Δl_f [Eq. (8) and (9)] and by an accurate evaluation of the forces, F (see p. 25), in the structural system through appropriate structural analysis.
2. The upper bound, UB, of the maximum joint closing obviously will depend on the coefficient of thermal expansion of the material of the frame, the maximum temperature change (i.e., the effective temperature increase $\Delta t_e = T_w - T_m$) that the structural frame is assumed to undergo, and the

effective length, L, of the structural segments converging at the joints. The effective length, L, can be computed utilizing the following empirical guidelines in conjunction with Figure 6:

a. If both the building segments converging on the joint have symmetrical stiffness, only one half of the dimensional change of each segment will affect the joint separation (Figure 6a), hence,
   $L = 1/2\ (L_1+L_2)$. (11)
b. If, however, either segment has one end substantially stiffer than the other, the dimensional change resulting from temperature fluctuation will be distributed unevenly between the two ends of such a segment with comparatively less deformation developing at the stiff end. In such cases,

$$L = 1/2\ (KL_1+L_2), \quad (12)$$

where K = 1.5 (i.e., the length of the unsymmetrically stiff segment will be increased by 50 percent if the stiff end is farthest away from the joint; see Figure 6b) or K = 0.67 (i.e., the length of the unsymmetrically stiff end will be decreased by 33 percent if the stiff end is the one abutting the joint; see Figure 6c).

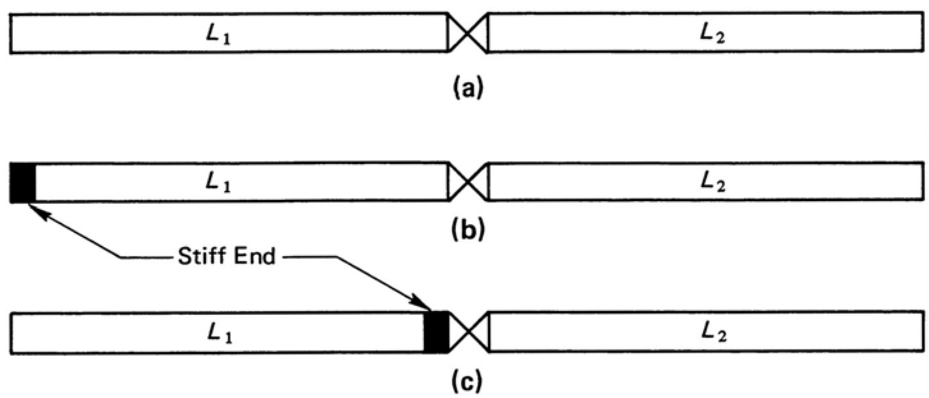


FIGURE 6 Computation of effective length L of building segments adjacent to the expansion joint: (a) building segments with symmetrical stiffness, $L = 1/2\ (L1+L_2)$; (b) one segment with unsymmetrical stiffness and the stiff end farthest from the joint, $L = 1/2\ (1.5L1+L_2)$; (c) one segment with unsymmetrical stiffness and the stiff end abutting the joint, L = 1/2 (0.67L1+L2).

The coefficient of thermal expansion of concrete and steel (the principal materials used for buildings with column-and-beam frames) can be considered approximately the same and equal to $6 \cdot 10^{-6}$ per degree Fahrenheit. The upper bound, UB, of the maximum joint closing can be computed from the expression:

$$UB = (6 \cdot 10^{-6})\, \Delta t_e \cdot L, \quad (13)$$

where Δt_e and L are as previously defined.

3. The actual width of the expansion joint must be greater than the UB to provide for construction tolerances and for the width and compressibility or expandability of the joint filler. The UB is likely to develop in those buildings that are not temperature controlled; for this condition, a joint width equal to twice the UB probably would be required. Because the maximum horizontal movement in temperature-controlled buildings is expected to be lower than that in noncontrolled buildings, the joint width can be narrower. Joint widths equal to 1.7 times the UB for buildings heated but not air conditioned and equal to 1.4 times the UB for buildings both heated and air conditioned should be sufficient.
4. For buildings with exterior bearing walls of continuous clay masonry the required joint width, W, can be determined by the expression:

   $$W = C_1 \cdot L\, (50°F + \Delta t_e)\, (4 \cdot 10^{-6}), \quad (14)$$

   where C_1 = 2.0 for buildings with no heat control, 1.7 for buildings heated but not air conditioned, and 1.4 for buildings both heated and air conditioned and Δt_e and L are as previously defined.

   In this expression $4 \cdot 10^{-6}$ is a coefficient approximateing the coefficient of thermal expansion of clay masonry. The term 50 °F in the factor $(50°F+\Delta t_e)$ represents a temperature equivalent to the dimensional changes resulting from potential of swelling of clay masonry under moisture conditions. Finally, the coefficient C_1 is intended to provide for construction tolerances, compressibility and expandability of the joint filler, and the dampening effects on the effective Δt_e of temperature control.

   The values for C_1, which are based on the judgment of the Committee members, are comparable to the correction factors recommended for use with Figure 1 when buildings are provided with temperature control. The rationale for the values is the same as for the Figure 1 correction factors.
5. Notwithstanding the above procedures, practical limits on the width of an expansion joint need to be adopted. It seems reasonable that, in general, an expansion joint should not be narrower than 1 inch. On the other hand, an expansion joint that, according to the computations above, requires greater than 2 inches of width should be specially designed to

ensure that these relatively large dimensional changes can take place without any loss of building serviceability. During architectural design and filler material selection, care must be taken to ensure that the functional and aesthetic requirements of the building are satisfactorily met and that the joint will be sufficiently flexible to guarantee durable and trouble-free operation.

6. It is necessary that an expansion joint extend all the way to the footing because, as is indicated by the analytical studies conducted on two-dimensional frames, a large percentage (on the order of 75 percent) of the maximum dimensional change due to temperature fluctuation develops in the lowest story of a structure and almost the maximum change develops in all the stories above.
7. An expansion joint requires protection from potential accumulation of foreign material or debris that could interfere with the proper functioning of the two parts of the joint. The joint should be designed in such a way that it can be maintained and inspected without difficulty to ensure that it remains effective.

D. AREAS OF FUTURE RESEARCH

For convenience, the scope of the Committee's study was limited to expansion joints that separate structural frames of buildings in order to relieve excessive temperature-induced stresses. The practices and procedures suggested herein are considered to be sound and should guide the designer in producing a more efficient building system than in the past. Also, they have been based for the most part on experience and educated judgment. Temperature fluctuations also effect dimensional changes in the vertical direction and the performance of the nonstructural building components; while such effects are not considered in this report, they cannot be ignored during design.

Execution of the most efficient design with respect to the total effects of temperature changes on building performance requires criteria developed on a data base more technically sound than exists at present. Thus, research should be undertaken immediately to provide urgently needed information and data that:

1. Reflect building damage directly attributable to temperature fluctuation.
2. Permit the correlation of ambient temperature with temperatures of building components (structural and nonstructural) at the periphery and within buildings for different building types and materials.
3. Permit the correlation of ambient temperature fluctuations with temperature gradients existing within building components under different conditions of exposure and insulation of these components.

Also needed are analytic and experimental investigations that will lead to the correlation of stresses in the various building components with the different patterns of temperature fluctuations and gradients and with the different types of assembly component (connectors).

Buildings supported on masonry walls require special examination since effects of temperature changes on the performance of such buildings will vary according to the type of masonry material or combinations of material used. Each type and combination should be investigated with respect to construction details, connections of walls to horizontal and vertical components (roofs, floors, walls, and partitions at right angles), optimal spacing of joints, and extent of joints.

APPENDIX A

COMPUTER PRINTOUT OF AN ELASTIC ANALYSIS

This section gives an example of a computer printout of an analysis for a frame with the extreme columns on one end substantially stiffer than the rest of the columns (see Figure A-1). This example corresponds to analysis M-1 presented in Table 2. The frame was analyzed for a 100 °F uniform temperature increase and for a coefficient of thermal expansion equal to 6 per million degrees Fahrenheit.

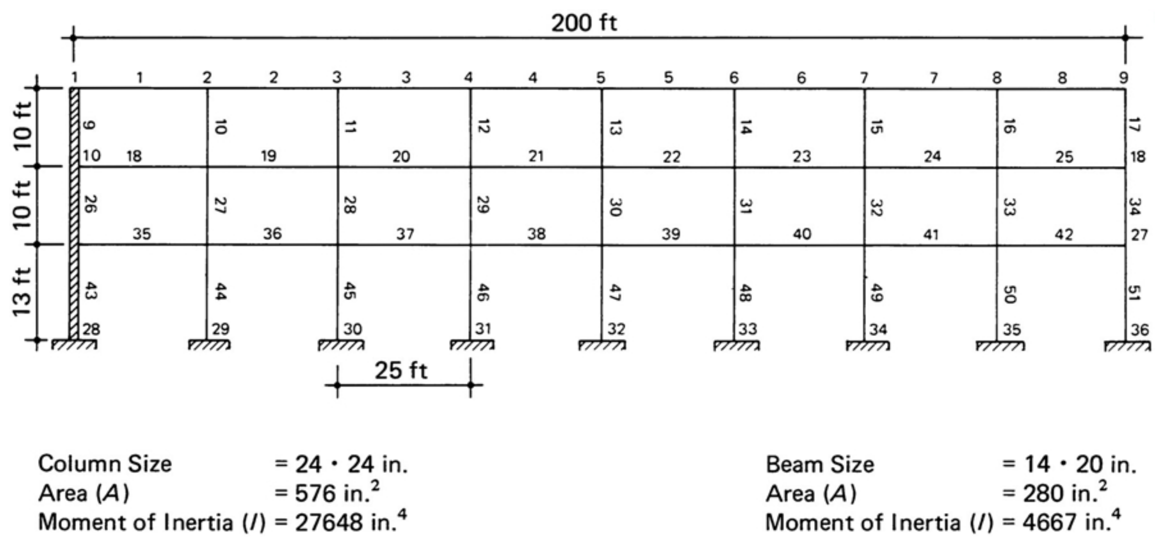


FIGURE A-1 Elastic analysis of a frame with columns at one end stiffer than the rest of the columns.

Analysis M-1

INPUT DATA

PLANE FRAME BY STIFFNESS EQTS M.SOTERIADES 1969

STRUCTURE NO. 2 LOADING NO. 1

STRUCTURE PARAMETERS*

M	NJ	NR	NRJ	E
51	36	27	9	3000000.

JOINT COORDINATES AND RESTRAINTS

JOINT	X	Y	X-RESTR.	Y-RESTR.	Z-RESTR.
1	0.00	396.00	0	0	0
2	300.00	396.00	0	0	0
3	600.00	396.00	0	0	0
4	900.00	396.00	0	0	0
5	1200.00	396.00	0	0	0
6	1500.00	396.00	0	0	0
7	1800.00	396.00	0	0	0
8	2100.00	396.00	0	0	0
9	2400.00	396.00	0	0	0
10	0.00	276.00	0	0	0
11	300.00	276.00	0	0	0
12	600.00	276.00	0	0	0
13	900.00	276.00	0	0	0
14	1200.00	276.00	0	0	0
15	1500.00	276.00	0	0	0
16	1800.00	276.00	0	0	0
17	2100.00	276.00	0	0	0
18	2400.00	276.00	0	0	0
19	0.00	156.00	0	0	0
20	300.00	156.00	0	0	0
21	600.00	156.00	0	0	0
22	900.00	156.00	0	0	0
23	1200.00	156.00	0	0	0
24	1500.00	156.00	0	0	0
25	1800.00	156.00	0	0	0
26	2100.00	156.00	0	0	0
27	2400.00	156.00	0	0	0
28	0.00	0.00	1	1	1
29	300.00	0.00	1	1	1
30	600.00	0.00	1	1	1
31	900.00	0.00	1	1	1
32	1200.00	0.00	1	1	1
33	1500.00	0.00	1	1	1
34	1800.00	0.00	1	1	1
35	2100.00	0.00	1	1	1
36	2400.00	0.00	1	1	1

*Legend:

M = No. of members in the frame
NJ = No. of joints in the frame
NR = No. of support restraints
NRJ = No. of restrained supports
E = Modulus of elasticity (lb/in.^{2})
JJ = Left end of member
JK = Right end of member
CX = Directional cosine (x)
CY = Directional cosine (y)
R = End restraint index (zero = fixed at both ends)
W = Dead load

Notes:

1. Coordinates x and y are entered in inches and relate to a Cartesian system positioned on joint 28.
2. Member cross-sectional areas A are entered in in.^{2}
3. Moments of inertia of members I are entered in in.^{4}
4. Member lengths are given in inches
5. In general, all dimensions used in input and output are exclusively in inches and pounds

MEMBER PROPERTIES*

I	JJ	JK	AREA	INERTIA	LENGTH	CX	CY	R	W
1	1	2	280.00	4667.00	300.00	1.000	0.000	0	0.000
2	2	3	280.00	4667.00	300.00	1.000	0.000	0	0.000
3	3	4	280.00	4667.00	300.00	1.000	0.000	0	0.000
4	4	5	280.00	4667.00	300.00	1.000	0.000	0	0.000
5	5	6	280.00	4667.00	300.00	1.000	0.000	0	0.000
6	6	7	280.00	4667.00	300.00	1.000	0.000	0	0.000
7	7	8	280.00	4667.00	300.00	1.000	0.000	0	0.000
8	8	9	280.00	4667.00	300.00	1.000	0.000	0	0.000
9	1	10	576.00	0.00	120.00	0.000	-1.000	0	0.000
10	2	11	576.00	27648.00	120.00	0.000	-1.000	0	0.000
11	3	12	576.00	27648.00	120.00	0.000	-1.000	0	0.000
12	4	13	576.00	27648.00	120.00	0.000	-1.000	0	0.000
13	5	14	576.00	27648.00	120.00	0.000	-1.000	0	0.000
14	6	15	576.00	27648.00	120.00	0.000	-1.000	0	0.000
15	7	16	576.00	27648.00	120.00	0.000	-1.000	0	0.000
16	8	17	576.00	27648.00	120.00	0.000	-1.000	0	0.000
17	9	18	576.00	0.00	120.00	0.000	-1.000	0	0.000
18	10	11	280.00	4667.00	300.00	1.000	0.000	0	0.000
19	11	12	280.00	4667.00	300.00	1.000	0.000	0	0.000
20	12	13	280.00	4667.00	300.00	1.000	0.000	0	0.000
21	13	14	280.00	4667.00	300.00	1.000	0.000	0	0.000
22	14	15	280.00	4667.00	300.00	1.000	0.000	0	0.000
23	15	16	280.00	4667.00	300.00	1.000	0.000	0	0.000
24	16	17	280.00	4667.00	300.00	1.000	0.000	0	0.000
25	17	18	280.00	4667.00	300.00	1.000	0.000	0	0.000
26	10	19	576.00	0.00	120.00	0.000	-1.000	0	0.000
27	11	20	576.00	27648.00	120.00	0.000	-1.000	0	0.000
28	12	21	576.00	27648.00	120.00	0.000	-1.000	0	0.000
29	13	22	576.00	27648.00	120.00	0.000	-1.000	0	0.000
30	14	23	576.00	27648.00	120.00	0.000	-1.000	0	0.000
31	15	24	576.00	27648.00	120.00	0.000	-1.000	0	0.000
32	16	25	576.00	27648.00	120.00	0.000	-1.000	0	0.000
33	17	26	576.00	27648.00	120.00	0.000	-1.000	0	0.000
34	18	27	576.00	0.00	120.00	0.000	-1.000	0	0.000
35	19	20	280.00	4667.00	300.00	1.000	0.000	0	0.000
36	20	21	280.00	4667.00	300.00	1.000	0.000	0	0.000
37	21	22	280.00	4667.00	300.00	1.000	0.000	0	0.000
38	22	23	280.00	4667.00	300.00	1.000	0.000	0	0.000
39	23	24	280.00	4667.00	300.00	1.000	0.000	0	0.000
40	24	25	280.00	4667.00	300.00	1.000	0.000	0	0.000
41	25	26	280.00	4667.00	300.00	1.000	0.000	0	0.000
42	26	27	280.00	4667.00	300.00	1.000	0.000	0	0.000
43	19	28	576.00	0.00	156.00	0.000	-1.000	0	0.000
44	20	29	576.00	27648.00	156.00	0.000	-1.000	0	0.000
45	21	30	576.00	27648.00	156.00	0.000	-1.000	0	0.000
46	22	31	576.00	27648.00	156.00	0.000	-1.000	0	0.000
47	23	32	576.00	27648.00	156.00	0.000	-1.000	0	0.000
48	24	33	576.00	27648.00	156.00	0.000	-1.000	0	0.000
49	25	34	576.00	27648.00	156.00	0.000	-1.000	0	0.000
50	26	35	576.00	27648.00	156.00	0.000	-1.000	0	0.000
51	27	36	576.00	0.00	156.00	0.000	-1.000	0	0.000

****

LOAD DATA

LOADED JOINTS= 0

LOADED MEMBS.= 0 (DISTR. L= 0 CONCEN. L= 0 OTHER L= 0)

UNIFORM TEMP. CHANGE=100 DEGREES
COEFF. OF THERMAL EXPANSION= 6.000 PER MILLION DEGREES

*See Legend, p. 34.

OUTPUT DATA

JOINT DISPLACEMENTS

JOINT	DIRECTION-X	DIRECTION-Y	DIRECTION-Z
27	0.6006209E 00	0.9349361E-01	0.1217476E-02
26	0.4206209E 00	0.9335021E-01	-0.2433518E-02
25	0.2678654E 00	0.9373797E-01	-0.1578297E-02
24	0.1304534E 00	0.9374416E-01	-0.7919014E-03
23	-0.1678797E-07	0.9374802E-01	0.1405881E-09
22	-0.1304534E 00	0.9374417E-01	0.7919017E-03
21	-0.2678654E 00	0.9373797E-01	0.1578298E-02
20	-0.4206209E 00	0.9335021E-01	0.2433519E-02
19	-0.6006209E 00	0.9349361E-01	-0.1217476E-02
18	0.7317835E 00	0.1654906E 00	0.1052131E-03
17	0.5517835E 00	0.1653390E 00	-0.2089101E-03
16	0.3677876E 00	0.1657378E 00	-0.2320321E-03
15	0.1836831E 00	0.1657527E 00	-0.1379756E-03
14	-0.3231054E-07	0.1657592E 00	0.1037993E-09
13	-0.1836831E 00	0.1657527E 00	0.1379758E-03
12	-0.3677877E 00	0.1657378E 00	0.2320323E-03
11	-0.5517836E 00	0.1653390E 00	0.2089104E-03
10	-0.7317836E 00	0.1654906E 00	-0.1052132E-03
9	0.7292213E 00	0.2374945E 00	-0.5875955E-04
8	0.5492213E 00	0.2373495E 00	0.1189688E-03
7	0.3675841E 00	0.2377333E 00	0.9926945E-04
6	0.1842663E 00	0.2377463E 00	0.5139868E-04
5	-0.4258106E-07	0.2377525E 00	0.6599168E-10
4	-0.1842664E 00	0.2377463E 00	-0.5139855E-04
3	-0.3675841E 00	0.2377333E 00	-0.9926933E-04
2	-0.5492214E 00	0.2373495E 00	-0.1189687E-03
1	-0.7292214E 00	0.2374945E 00	0.5875952E-04

MEMBER END ACTIONS*

I	NLF	VLF	MLF	NRT	VRT	MRT	
1	-0.488281E-03	-0.552971E 02	-0.224113E-03	0.488281E-03	0.552971E 02	-0.165891E 05	
2	-0.458420E 04	-0.206091E 03	-0.318331E 05	0.458420E 04	0.206091E 03	-0.299943E 05	
3	-0.928973E 04	-0.140714E 03	-0.233412E 05	0.928973E 04	0.140714E 03	-0.188730E 05	
4	-0.119458E 05	-0.480141E 02	-0.960090E 04	0.119458E 05	0.480141E 02	-0.480335E 04	
5	-0.119458E 05	0.480144E 02	0.480339E 04	0.119458E 05	-0.480144E 02	0.960093E 04	Girders
6	-0.928972E 04	0.140714E 03	0.188730E 05	0.928972E 04	-0.140714E 03	0.233413E 05	
7	-0.458420E 04	0.206091E 03	0.299944E 05	0.458420E 04	-0.206091E 03	0.318331E 05	
8	0.976562E-03	0.552972E 02	0.165891E 05	-0.976562E-03	-0.552972E 02	-0.105738E-02	
9	-0.552998E 02	0.000000E 00	0.000000E 00	0.552998E 02	0.000000E 00	0.000000E 00	
10	-0.150799E 03	0.458420E 04	0.484222E 05	0.150799E 03	-0.458420E 04	0.501682E 06	
11	0.653798E 02	0.470552E 04	0.533356E 05	-0.653798E 02	-0.470552E 04	0.511327E 06	
12	0.927011E 02	0.265615E 04	0.284739E 05	-0.927011E 02	-0.265615E 04	0.290265E 06	
13	0.960312E 02	-0.478429E-04	-0.290032E-01	-0.960312E 02	0.478429E-04	0.232620E-01	Columns
14	0.927011E 02	-0.265615E 04	-0.284739E 05	-0.927011E 02	0.265615E 04	-0.290265E 06	
15	0.653730E 02	-0.470552E 04	-0.533357E 05	-0.653730E 02	0.470552E 04	-0.511327E 06	
16	-0.150789E 03	-0.458420E 04	-0.484223E 05	0.150789E 03	0.458420E 04	-0.501682E 06	
17	-0.552958E 02	0.000000E 00	0.000000E 00	0.552958E 02	0.000000E 00	0.000000E 00	
18	0.976562E-03	0.977343E 02	0.203728E-03	-0.976562E-03	-0.977343E 02	0.293203E 05	
19	-0.111884E 05	0.409094E 03	0.602850E 05	0.111884E 05	-0.409094E 03	0.624432E 05	
20	-0.114927E 05	0.345272E 03	0.561805E 05	0.114927E 05	-0.345272E 03	0.474013E 05	
21	-0.103128E 05	0.128746E 03	0.257513E 05	0.103128E 05	-0.128746E 03	0.128726E 05	Girders
22	-0.103128E 05	-0.128746E 03	-0.128725E 05	0.103128E 05	0.128746E 03	-0.257512E 05	
23	-0.114927E 05	-0.345272E 03	-0.474012E 05	0.114927E 05	0.345272E 03	-0.561804E 05	
24	-0.111884E 05	-0.409093E 03	-0.624431E 05	0.111884E 05	0.409093E 03	-0.602849E 05	
25	0.341796E-02	-0.977342E 02	-0.293202E 05	-0.341796E-02	0.977342E 02	0.212192E-04	
26	0.424482E 02	0.000000E 00	0.000000E 00	-0.424482E 02	0.000000E 00	0.000000E 00	
27	0.160576E 03	0.157726E 05	-0.591287E 06	-0.160576E 03	-0.157726E 05	0.248401E 07	
28	0.158398E 01	0.500980E 04	-0.629950E 06	-0.158398E 01	-0.500980E 04	0.123112E 07	
29	-0.123802E 03	0.147626E 04	-0.363417E 06	0.123802E 03	-0.147626E 04	0.540569E 06	
30	-0.161437E 03	-0.494965E-03	-0.551263E-01	0.161437E 03	0.494965E-03	-0.426955E-02	Columns
31	-0.123804E 03	-0.147626E 04	0.363417E 06	0.123804E 03	0.147626E 04	-0.540569E 06	
32	0.157812E 01	-0.500980E 04	0.629950E 06	-0.157812E 01	0.500980E 04	-0.123112E 07	
33	0.160594E 03	-0.157726E 05	0.591287E 06	-0.160594E 03	0.157726E 05	-0.248401E 07	
34	0.424541E 02	0.000000E 00	0.000000E 00	-0.424541E 02	0.000000E 00	0.000000E 00	
35	-0.488281E-03	0.113594E 04	-0.516057E-03	0.488281E-03	-0.113594E 04	0.340783E 06	
36	0.762845E 05	0.374221E 04	0.601245E 06	-0.762845E 05	-0.374221E 04	0.521419E 06	
37	0.119246E 06	0.221230E 04	0.368547E 06	-0.119246E 06	-0.221230E 04	0.295144E 06	
38	0.138730E 06	0.739137E 03	0.147828E 06	-0.138730E 06	-0.739137E 03	0.739125E 05	Girders
39	0.138730E 06	-0.739136E 03	-0.739124E 05	-0.138730E 06	0.739136E 03	-0.147828E 06	
40	0.119246E 06	-0.221230E 04	-0.295144E 06	-0.119246E 06	0.221230E 04	-0.368546E 06	
41	0.762845E 05	-0.374221E 04	-0.521419E 06	-0.762845E 05	0.374221E 04	-0.601245E 06	
42	0.195312E-02	-0.113594E 04	-0.340783E 06	-0.195312E-02	0.113594E 04	0.230908E-03	
43	0.117840E 04	0.000000E 00	0.000000E 00	-0.117840E 04	0.000000E 00	0.000000E 00	
44	0.276685E 04	-0.605118E 05	-0.342604E 07	-0.276685E 04	0.605118E 05	-0.601380E 07	
45	-0.152831E 04	-0.379520E 05	-0.212109E 07	0.152831E 04	0.379520E 05	-0.379943E 07	
46	-0.159696E 04	-0.180075E 05	-0.983542E 06	0.159696E 04	0.180075E 05	-0.182563E 07	
47	-0.163970E 04	-0.152641E-02	-0.443106E-01	0.163970E 04	0.152641E-02	-0.193809E 00	Columns
48	-0.159695E 04	0.180075E 05	0.983542E 06	0.159695E 04	-0.180075E 05	0.182563E 07	
49	-0.152831E 04	0.379520E 05	0.212109E 07	0.152831E 04	-0.379520E 05	0.379943E 07	
50	0.276688E 04	0.605118E 05	0.342604E 07	-0.276688E 04	-0.605118E 05	0.601380E 07	
51	0.117841E 04	0.000000E 00	0.000000E 00	-0.117841E 04	0.000000E 00	0.000000E 00	

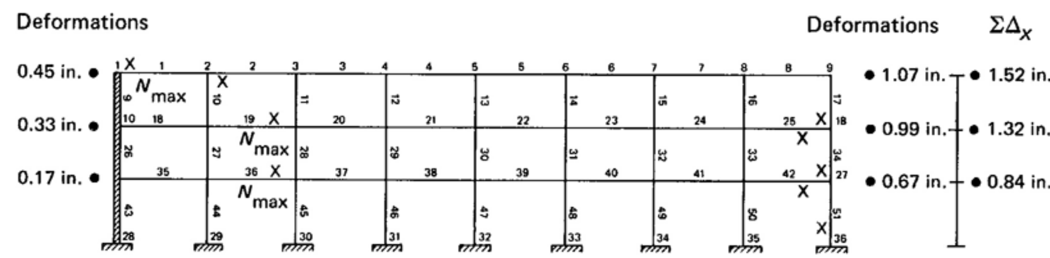
*Legend:

NLF = Axial force: left end (lb)
VLF = Shear force: left end (lb)
MLF = Moment: left end (lb)

NRT = Axial force: right end (lb)
VRT = Shear force: right end (lb)
MRT = Moment: right end (lb)

SUPPORT REACTIONS

JOINT	DIRECTION-X	DIRECTION-Y	DIRECTION-Z
28	0.0000000E 00	0.1178404E 04	0.0000000E 00
29	0.6051185E 05	0.2766857E 04	-0.6013807E 07
30	0.3795208E 05	-0.1528319E 04	-0.3799431E 07
31	0.1800757E 05	-0.1596960E 04	-0.1825638E 07
32	0.1526414E-02	-0.1639701E 04	-0.1938099E 00
33	-0.1800757E 05	-0.1596957E 04	0.1825638E 07
34	-0.3795207E 05	-0.1528318E 04	0.3799431E 07
35	-0.6051184E 05	0.2766884E 04	0.6013807E 07
36	0.0000000E 00	0.1178416E 04	0.0000000E 00



x Designates Location of Critical Stress (N, V, or M)

Uninhibited Motion $\Delta_o = \alpha \Delta_t L = \alpha \cdot 100^\circ \cdot L = 1.44$ in.

$\Sigma\Delta_x = 1.52$ in.

Critical Stress Components

Floor Level	Column		Girder	
	M_{max} (ft-kips)	V_{max} (kips)	M_{max} (ft-kips)	N_{max} (kips)
Base floor	740(51)[a]	83.2 (51)	∿100(42)	253(36)
Mid floor	245(34)	11.1(34)	∿30(25)	57.3(19)
Top floor	100(17)	9.4(17)	∿11(2)	46.8(1)

[a]Numbers in parenthesis refer to the number of the member.

APPENDIX B

TEMPERATURE DATA

The following tabulation presents mean construction season temperature (T_m) and extreme summer (T_w) and winter (T_c) temperature data for various localities in the United States. Most stations listed are located at airports; those identified as CO are city offices.

T_m = the mean temperature during the normal construction season in the locality of the building. For the purpose of this report the normal construction season for a locality is defined as that contiguous period in a year during which the minimum daily temperature equals or exceeds 32 °F.*

T_w = the temperature exceeded, on the average, only 1 percent of the time during the summer months of June through September in the locality of the building. (In a normal summer there would be approximately 30 hours at or above the design value.**)

T_c = the temperature equaled or exceeded, on the average, 99 percent of the time during the winter months of December, January, and February in the locality of the building. (In a normal winter there would be approximately 22 hours at or below this design value.**)

*These contiguous periods for each locality in the United States were obtained from the Decennial Census of United States Climate: Daily Normals of Temperature and Heating Degree Days (see reference on page 11) and the mean construction season temperature values T_m were computed (by Maj. T. E. Stanton of the USAF Environmental Technical Applications Center, Washington, D. C.) from the mean monthly temperatures extracted from the National Weather Services' Local Climatological Data Summaries for the stations. In a few cases other sources also were used.

**The T_w and T_c values are extracted from the ASHRAE Handbook of Fundamentals (1972), published by the American Society of Heating, Refrigerating and Air-Conditioning Engineers.

Station	Temperature (°F) T_w	T_m	T_c
Alabama			
Birmingham	97	63	19
Huntsville	97	61	13
Mobile (CO)	96	68	28
Montgomery	98	66	22
Alaska			
Anchorage	73	51	-25
Barrow	58	38	-45
Fairbanks	82	50	-53
Juneau	75	48	-7
Nome	66	45	-32
Arizona			
Flagstaff	84	58	0
Phoenix	108	70	31
Prescott	96	64	15
Tuscon	105	67	29
Winslow	97	67	9
Yuma	111	72	37
Arkansas			
Ft. Smith	101	65	15
Little Rock	99	65	19
Texarkana	99	65	22
California			
Bakersfield	103	65	31
Burbank	97	64	36
Eureka/Arcata	67	52	32
Fresno	101	63	28
Long Beach	87	63	41
Los Angeles	94	62	41
Oakland	85	57	35
Sacramento	100	60	30
San Diego	86	62	42
San Francisco	83	56	35
Santa Maria	85	57	32
Colorado			
Alamosa	84	60	-17
Colorado Springs	90	61	-1
Denver	92	62	-2
Grand Junction	96	64	8
Pueblo	96	64	-5
Connecticut			
Bridgeport	90	60	4
Hartford	90	61	1
New Haven	88	59	5
Delaware			
Wilmington	93	62	12
Florida			
Daytona Beach	94	70	32
Ft. Myers	94	74	38
Florida (Continued)			
Jacksonville	96	68	29
Key West	90	77	55
Lakeland (CO)	95	72	35
Miami	92	75	44
Miami Beach (CO)	91	75	45
Orlando	96	72	33
Pensacola (CO)	92	68	29
Tallahassee	96	68	25
Tampa	92	72	36
West Palm Beach	92	75	40
Georgia			
Athens	96	61	17
Atlanta	95	62	18
Augusta	98	64	20
Columbus	98	65	23
Macon	98	65	23
Rome	97	62	16
Savannah/Travis	96	67	24
Hawaii			
Hilo	85	73	59
Honolulu	87	76	60
Idaho			
Boise	96	61	4
Idaho Falls	91	61	-12
Lewiston	98	60	6
Pocatello	94	60	-8
Illinois			
Chicago	95	60	-3
Moline	94	63	-7
Peoria	94	61	-2
Rockford	92	62	-7
Springfield	95	62	-1
Indiana			
Evansville	96	65	6
Fort Wayne	93	62	0
Indianapolis	93	63	0
South Bend	92	61	-2
Iowa			
Burlington	95	64	-4
Des Moines	95	64	-7
Dubuque	62	63	-11
Sioux City	96	64	-10
Waterloo	91	63	-12
Kansas			
Dodge City	99	64	3
Goodland	99	65	-2
Topeka	99	69	3
Wichita	102	68	5

Station	Temperature (°F)		
	T_w	T_m	T_c
Kentucky			
Covington	93	63	3
Lexington	94	63	6
Louisville	96	64	8
Louisiana			
Baton Rouge	96	68	25
Lake Charles	95	68	29
New Orleans	93	69	32
Shreveport	99	66	22
Maine			
Caribou	85	56	-18
Portland	88	58	-5
Maryland			
Baltimore	94	63	12
Frederick	94	63	7
Massachusetts			
Boston	91	58	6
Pittsfield	86	58	-5
Worcester	89	58	-3
Michigan			
Alpena	87	57	-5
Detroit-Metropolitan	92	58	4
Escanaba	82	55	-7
Flint	89	60	-1
Grand Rapids	91	62	2
Lansing	89	59	2
Marquette	88	55	-8
Muskegon	87	59	4
Sault Ste Marie	83	55	-12
Minnesota			
Duluth	85	55	-19
International Falls	86	57	-29
Minneapolis/St. Paul	92	62	-14
Rochester	90	60	-17
St. Cloud	90	60	-20
Mississippi			
Jackson	98	66	21
Meridian	97	65	20
Vicksburg (CO)	97	66	23
Missouri			
Columbia	97	65	2
Kansas City	100	65	4
St. Joseph	97	66	-1
St. Louis	98	65	4
Springfield	97	64	5

Station	Temperature (°F)		
	T_w	T_m	T_c
Montana			
Billings	94	60	-10
Glasgow	96	60	-25
Great Falls	91	58	-20
Havre	91	58	-22
Helena	90	58	-17
Kalispell	88	56	-7
Miles City	97	62	-19
Missoula	92	58	-7
Nebraska			
Grand Island	98	65	-6
Lincoln (CO)	100	64	-4
Norfolk	97	64	-11
North Platte	97	64	-6
Omaha	97	64	-5
Scottsbluff	96	62	-8
Nevada			
Elko	94	61	-13
Ely	90	59	-6
Las Vegas	108	66	23
Reno	95	62	2
Winnemucca	97	63	1
New Hampshire			
Concord	91	60	-11
New Jersey			
Atlantic City	91	61	14
Newark	94	62	11
Trenton (CO)	92	61	12
New Mexico			
Albuquerque	96	64	14
Raton	92	64	-2
Roswell	101	70	16
New York			
Albany	91	61	-5
Binghampton (CO)	91	67	-2
Buffalo	88	59	3
New York	94	59	11
Rochester	91	59	2
Syracuse	90	59	-2
North Carolina			
Asheville	91	60	13
Charlotte	96	60	18
Greensboro	94	64	14
Raleigh/Durham	95	62	16
Wilmington	93	63	23
Winston/Salem	94	63	14

Station	Temperature (°F) T_w	T_m	T_c
North Dakota			
Bismarck	95	60	-24
Devils Lake	93	58	-23
Fargo	92	59	-22
Minot	91		-24
Williston	94	59	-21
Ohio			
Akron/Canton	89	60	1
Cincinnati (CO)	94	62	8
Cleveland	91	61	2
Columbus	92	61	2
Dayton	92	61	0
Mansfield	91	61	1
Sandusky (CO)	92	60	4
Toledo	92	61	1
Youngstown	89	59	1
Oklahoma			
Oklahoma City	100	64	11
Tulsa	102	65	12
Oregon			
Astoria	79	50	27
Eugene	91	52	22
Medford	98	56	21
Pendleton	97	58	3
Portland	91	52	21
Roseburg	93	54	25
Salem	92	52	21
Pennsylvania			
Allentown	92	61	3
Erie	88	59	7
Harrisburg	92	61	9
Philadelphia	93	63	11
Pittsburgh	90	63	5
Reading (CO)	92	61	6
Scranton/Wilkes-Barre	89	61	2
Williamsport	91	61	1
Rhode Island			
Providence	89	60	6
South Carolina			
Charleston	95	66	23
Columbia	98	64	20
Florence	96	64	21
Greenville	95	61	19
Spartanburg	95	60	18
South Dakota			
Huron	97	62	-16
Rapid City	96	61	-9
Souix Falls	95	62	-14

Station	Temperature (°F) T_w	T_m	T_c
Tennessee			
Bristol/Tri City	92	63	11
Chattanooga	97	60	15
Knoxville	95	60	13
Memphis	98	62	17
Nashville	97	62	12
Texas			
Abilene	101	65	17
Amarillo	98	66	8
Austin	101	68	25
Brownsville	94	74	36
Corpus Christi	95	71	32
Dallas	101	66	19
El Paso	100	65	21
Fort Worth	102	66	20
Galveston	91	70	32
Houston	96	68	28
Laredo AFB	103	74	32
Lubbock	99	67	11
Midland	100	66	19
Port Arthur	94	69	29
San Angelo	101	65	20
San Antonio	99	69	25
Victoria	98	71	28
Waco	101	67	21
Wichita Falls	103	66	15
Utah			
Salt Lake City	97	63	5
Vermont			
Burlington	88	57	-12
Virginia			
Lynchburg	94	62	15
Norfolk	94	60	20
Richmond	96	64	14
Roanoke	94	63	15
Washington, D. C.			
National Airport	94	63	16
Washington			
Olympia	85	51	21
Seattle	85	51	20
Spokane	93	58	-2
Walla Walla	98	57	12
Yakima	94	62	6
West Virginia			
Charleston	92	63	9
Huntington (CO)	95	63	10
Parkersburg (CO)	93	62	8

Wisconsin			
Green Bay	88	59	`12
La Crosse	90	62	`12
Madison	92	61	`9
Milwaukee	90	60	`6
Wyoming			
Casper	92	59	`11
Cheyenne	89	58	`6
Lander	92	58	`16
Sheridan	95	59	`12